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Full one-loop electroweak corrections to $e^+e^- \to 3$ jets at linear colliders

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Summary. — We describe the impact of the full one-loop electroweak terms of $\mathcal{O}(\alpha_S\alpha_{EM}^3)$ entering the electron-positron into three-jet cross-section from $\sqrt{s} = M_Z$ to TeV scale energies. We include both factorisable and non-factorisable virtual corrections and photon bremsstrahlung. Their importance for the measurement of α_S from jet rates and shape variables is explained qualitatively and illustrated quantitatively, also in presence of b-tagging.

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1. – Introduction

A peculiar feature distinguishing strong (QCD) and electroweak (EW) effects in higher orders is that the latter are enhanced by (Sudakov) double logarithmic factors, $\ln^2(\frac{s}{M_W^2})$, which, unlike in the former, do not cancel for 'infrared-safe' observables [1, 2, 3, 4]. The origin of these 'double logs' is well understood. It is due to a lack of the Kinoshita-Lee-Nauenberg (KLN) [5] type cancellations of Infra-Red (IR) – both soft and collinear – virtual and real emission in higher order contributions originating from $W^\pm$ (and, possibly, Z) exchange. This is in turn a consequence of the violation of the Bloch-Nordsieck theorem [6] in non-Abelian theories [7]. The problem is in principle present also in QCD. In practice, however, it has no observable consequences, because of the final averaging of the colour degrees of freedom of partons. This does not occur in the EW case, where the initial state has a non-Abelian charge, dictated by the given collider beam configuration, such as in e^+e^- collisions.

These logarithmic corrections are finite (unlike in QCD), as the masses of the weak gauge bosons provide a physical cut-off for $W^\pm$ and Z emission. Hence, for typical experimental resolutions, softly and collinearly emitted weak bosons need not be included in the production cross-section and one can restrict oneself to the calculation of weak effects originating from virtual corrections and affecting a purely hadronic final state.

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Besides, these contributions can be isolated in a gauge-invariant manner from electromagnetic (EM) effects [3], at least in some specific cases, and therefore may or may not be included in the calculation, depending on the observable being studied. As for purely EM effects, since the (infinite) IR real photon emission cannot be resolved experimentally, this ought to be combined with the (also infinite) virtual one, through the same order, to recover a finite result, which is however not doubly logarithmically enhanced (as QED is an Abelian theory).

In view of all this, it becomes of crucial importance to assess the quantitative relevance of such EW corrections affecting, in particular, key QCD processes studied at past, present and future colliders, such as $e^+e^- \to 3$ jets.

2. – Calculation

In Ref. [8], we calculated the full one-loop EW effects entering three-jet production in e^+e^- annihilation at any collider energy via the subprocesses $e^+e^- \to \gamma^*, Z \to \bar{q}qg$. Ref. [9] tackled part of these, restricted to the case of $W^\pm$ and Z (but not γ) exchange and when the higher order effects arise only from initial or final state interactions (the so-called 'factorisable' corrections). The remainder, 'non-factorisable' corrections, while being typically small at $\sqrt{s} = M_Z$, are expected to play a quantitatively relevant role as $\sqrt{s}$ grows larger. We improved on the results of Ref. [9] in two respects: (i) we include now all the non-factorisable terms; (ii) we also incorporate previously neglected genuine QED corrections, including photon bremsstrahlung.

A more complete account of the corrections discussed here has recently appeared in Ref. [10].

Combining the enhancement associated with the weak Sudakov logarithms to the decrease of α_S with energy, in general, one expects one-loop EW effects to become comparable to QCD ones at future Linear Colliders (LCs) [11] running at TeV energy scales, like those available at an International Linear Collider (ILC) or the Compact LInear Collider (CLIC). In contrast, at the Z mass peak, where logarithmic enhancements are not effective, one-loop EW corrections are expected to appear at the percent level, hence being of limited relevance at LEP1 and SLC, where the final error on α_S is of the same order or larger, but of crucial importance at a GigaZ stage of a future LC [9], where the relative accuracy of α_S measurements is expected to be at the 0.1% level or better. Concerning higher order QCD effects, a great deal of effort has recently been devoted to evaluate two-loop contributions to the three-jet process [12] while the one-loop QCD results have been known for quite some time [13].

In e^+e^- annihilations, the most important QCD quantity to be extracted from multi-jet events is α_S. The confrontation of the measured value of the strong coupling constant with that predicted by the theory through the renormalisation group evolution is an important test of the Standard Model (SM). Alternatively, it may be an indication of new physics, when its typical mass scale is larger than the collider energy, so that the new particles cannot be produced as 'real' detectable states but may manifest themselves through 'virtual' effects. Not only jet rates, but also jet shape observables would be affected.

The detailed discussion of the calculation can be found in Ref. [8]. Here, for the sake of completeness, we mention that the calculation of virtual corrections is performed in the 't Hooft-Feynmann gauge. It is also worth mentioning that initial state electron-positron polarisations are retained and it is possible to study EW effects in presence of polarised

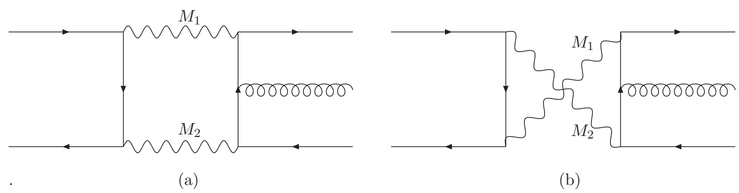

Fig. 1. – Pentagon graphs

incoming beams. For genuinely weak interaction corrections, this is of particular interest, since such corrections violate parity conservation.

In the calculation IR divergences are regulated by means of a small photon mass λ, both in virtual and real QED corrections. The independence of the final results from the photon mass has been successfully checked.

A new feature of this calculation is the occurrence of pentagon graphs, as those shown in Fig. 1. We have handled these in two separate ways (with two independently developed codes), in order to check for possible numerical instabilities, finding good agreement.

The collinear QED divergence gives rise to a large logarithm ($\ln(s/m_f^2)$), which is associated with the Initial State Radiation (ISR) induced by the incoming electrons and positrons. In the case of electron-positron colliders this large correction is always present and it is universal to all processes. For sensible numerical results, it has to be accounted for to all orders of perturbation theory, e.g., within the so-called electron/positron structure function formalism [14], which automatically resums in QED all Leading Logarithmic (LL) terms. In Ref. [15] a method of combining consistently resummed LL calculations with exact $\mathcal{O}(\alpha_{\rm EM})$ ones has been devised both in additive and factorisable form. Here, we adopted the additive approach.

In order to integrate over the phase-space, the width, Γ_Z, of the Z boson has been included in the propagator. For consistency, this means that the same width has to be included in the Z propagator for the virtual corrections. The essential ingredient for the evaluation of virtual corrections is the ability to compute one-loop integrals with complex internal masses. We implemented the general expression for the scalar four-point function of Ref. [16], valid also for complex masses. Particular attention has been devoted to the occurrence of numerical instabilities in certain regions of phase space because of strong cancellations.

We have neglected the masses of light quarks throughout. However, in the case in which the final state contains a $b\bar{b}$ pair, whenever there is a $W^\pm$ boson in the virtual loops, account had to be taken of the mass of the top (anti)quark. We are therefore in a position to present the results for such 'b-jets' separately, as reported in [17].

3. – Numerical results

The numerical results presented in this section are obtained considering a realistic experimental setup. The input parameters and the setup of the cuts is described in Ref. [8].

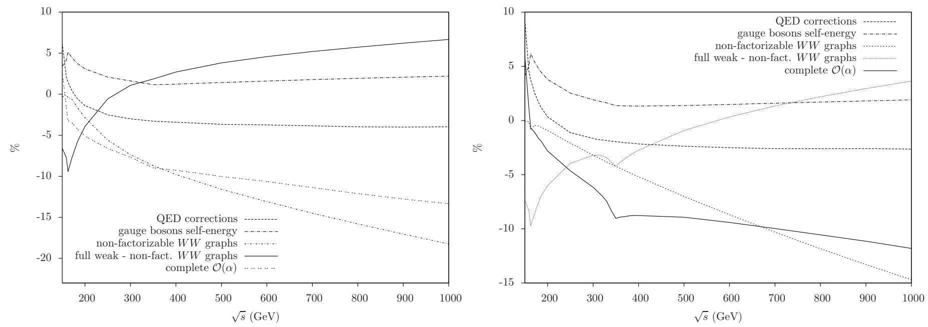

Fig. 2. – Relative effect on the integrated cross section due to different contributions to the order $\alpha \equiv \alpha_{\rm EM}$ correction, as a function of the CM energy. On the left the sample inclusive over the quark flavours is shown, on the right the b-jets subsample is considered.

A Cambridge jet algorithm is used to cluster parton momenta into jets. Finally, we sum over the final-state quarks, if not stated otherwise.

On the left of Fig. 2, the relative effects on the cross section induced by different contributions to the order $\alpha_{\rm S}\alpha_{\rm EM}^3$ correction are plotted as a function of the CM energy, in the range from 150 GeV to 1 TeV, when considering a sample summed over the quark flavours. The curves represent the effect of the QED corrections only, the effect of the gauge bosons self-energy corrections, the effect of the non-factorisable graphs with WW exchange, the effect of the weak corrections with the non-factorizing WW graphs removed (labelled as "full weak - non-fact WW graphs") and the total effect as the sum of the previous ones: the total effect is increasingly negative, reaching the -13% level at 1 TeV. It is worth mentioning that, as far as the non-factorisable WW corrections are concerned, in the case of d, s and b final-state quarks, only the direct diagrams are present due to charge conservation, while, for u and c quarks, only crossed diagrams are present, if the sum over initial- and final-state helicities is taken. In the case of ZZ exchange, all the graphs survive, giving rise to a cancellation at the leading-log level between direct and crossed diagrams, which does not occurr for WW exchange. Hence, the big negative correction is due to the presence of the WW non-factorisable graphs, which develop the aforementioned large Sudakov double logarithms in the high energy regime. In the right panel of Fig. 2, the corrections to the process $e^+e^- \to b\bar{b}g$ are shown, assuming that an efficient b-tagging is present.

We then show the impact of the EW corrections on some differential distributions of phenomenological interest. The plots show the tree-level contributions and the higher order corrections in three different contributions: the purely weak-interaction contribution (labelled "weak $\mathcal{O}(\alpha)$"), purely weak plus QED corrections, which are dominated by the above-mentioned ISR (labelled "exact $\mathcal{O}(\alpha)$"), and the weak plus electromagnetic correction in which the LL have been summed (labelled "exact $\mathcal{O}(\alpha)$ + h.o. LL"). The figures show in the upper panel the absolute distributions and in the lower panel the relative differences with respect to the tree-level rates.

In Fig. 3, the *thrust* event shape distribution is shown, in the form $(1-T)\frac{d\sigma}{dT}$. The T distribution is one of the key observables used for the measurement of $\alpha_{\rm S}$ in e^+e^- collisions [18]. It is worth noticing that while the purely weak corrections give an almost constant effect on the whole T range, the presence of the real bremsstrahlung gives a non

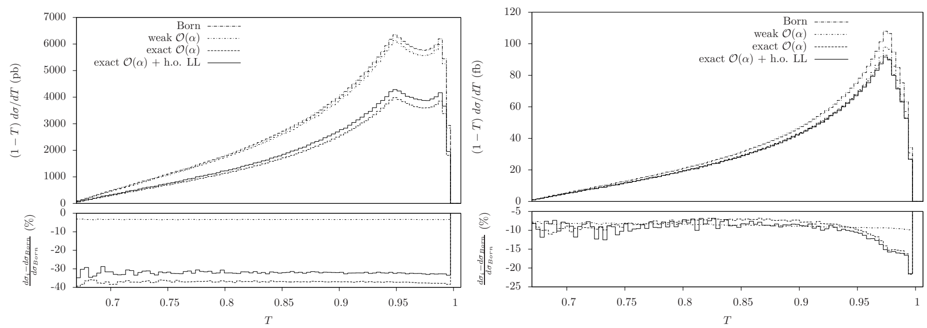

Fig. 3. – $(1-T)\frac{d\sigma}{dT}$ distribution at the Z peak (left) and at 1 TeV (right).

trivial effect in the region $T > 0.92$. In view of a precise measurement of α_S at a future LC, EW corrections can play an important role.

The ability to efficiently tag b-quark jets enables one to define observables in $b\bar{b}g$ final states which are not (easily) reconstructable in the case of the full three-jet sample. One example is the invariant mass of the $b\bar{b}$ pair, $M_{b\bar{b}}$, which we plot in Fig. 4. Here, the largest contribution to the total correction comes from QED ISR, primarily because of the radiative return phenomenon.

4. – Conclusions

In summary, we have shown the phenomenological relevance that the calculation up to $\mathcal{O}(\alpha_S\alpha_{EM}^3)$ can have in the study of (unflavoured) three-jet samples in e^+e^- annihilation, for all energies ranging from $\sqrt{s} = M_Z$ to 1 TeV. Not only inclusive jet rates are affected, but also more exclusive distributions, both global (like the event shape variables) and individual (like invariant mass) ones. Effects range from a few percent to several tens of percent, depending on the energy and the observable being studied, and we have shown cases where such higher-order contributions would impinge on the experimental measurements of jet quantities. Finally, notice that, depending on experimental procedures, a different normalisation of the distributions, like, e.g., the one adopted in Ref. [10], would

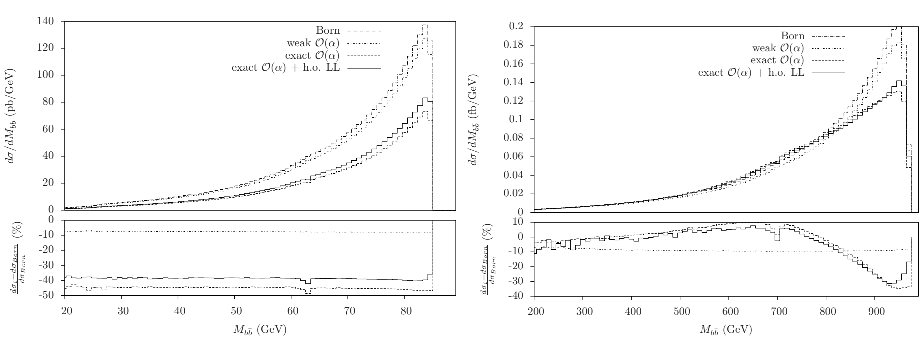

Fig. 4. – $b\bar{b}$ invariant mass distribution at the Z peak (left) and at 1 TeV (right).

lead to somewhat different corrections in general.

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REFERENCES

[1] M. Kuroda, G. Moultaka and D. Schildknecht, *Nucl. Phys. B*, **350** (1991) 25; G. Degrassi and A. Sirlin, *Phys. Rev. D*, **46** (1992) 3104; A. Denner, S. Dittmaier and R. Schuster, *Nucl. Phys. B*, **452** (1995) 80; A. Denner, S. Dittmaier and T. Hahn, *Phys. Rev. D*, **56** (1997) 117; A. Denner and T. Hahn, *Nucl. Phys. B*, **525** (1998) 27

[2] W. Beenakker, A. Denner, S. Dittmaier, R. Mertig and T. Sack, *Nucl. Phys. B*, **410** (1993) 245; *Phys. Lett. B*, **317** (1993) 622

[3] P. Ciafaloni and D. Comelli, *Phys. Lett. B*, **446** (1999) 278

[4] A. Denner and S. Pozzorini, *Eur. Phys. J. C*, **18** (2001) 461

[5] T. Kinoshita, *J. Math. Phys.*, **3** (1962) 650; T.D. Lee and M. Nauenberg,Phys. Rev.13319641549

[6] F. Bloch and A. Nordsieck, *Phys. Rev.*, **52** (1937) 54

[7] M. Ciafaloni, P. Ciafaloni and D. Comelli, *Phys. Rev. Lett.*, **84** (2000) 4810

[8] C. M. Carloni-Calame, S. Moretti, F. Piccinini and D. A. Ross, *JHEP*, **0903** (2009) 047

[9] E. Maina, S. Moretti, M.R. Nolten and D.A. Ross, arXiv:hep-ph/0407150; arXiv:hep-ph/0403269; *JHEP*, **0304** (2003) 056

[10] A. Denner, S. Dittmaier, T. Gehrmann and C. Kurz, arXiv:1003.0986; arXiv:1001.4467; *Phys. Lett. B*, **679** (2009) 219

[11] A. Djouadi, J. Lykken, K. Monig, Y. Okada, M.J. Oreglia and S. Yamashita, arXiv:0709.1893 (hep-ph); K. Abe et al. (The ACFA Linear Collider Working Group), arXiv:hep-ph/0109166; T. Abe et al. (The American Linear Collider Working Group), arXiv:hep-ex/0106055; arXiv:hep-ex/0106056; arXiv:hep-ex/0106057; arXiv:hep-ex/0106058; J.A. Aguilar-Saavedra et al. (The ECFA/DESY LC Physics Working Group),arXiv:hep-ph/0106315; G. Guignard (editor) (The CLIC Study Team), preprint CERN-2000-0082000

[12] A. Gehrmann-De Ridder, T. Gehrmann, E.W.N. Glover and G. Heinrich, *Phys. Rev. Lett*, **100** (2008) 172001; *JHEP*, **0712** (2007) 094; *JHEP*, **0711** (2007) 058; *Nucl. Phys. Proc. Suppl.*, **160** (2006) 190; A. Gehrmann-De Ridder, T. Gehrmann and E.W.N. Glover, *Nucl. Phys. Proc. Suppl.*, **135** (2004) 97; L.W. Garland, T. Gehrmann, E.W.N. Glover, A. Koukoutsakis and E. Remiddi, *Nucl. Phys. B*, **642** (2002) 227; *Nucl. Phys. B*, **627** (2002) 107; S. Weinzierl, arXiv:1001.1281 [hep-ph]; *Phys. Rev. D*, **80** (2009) 094018;*JHEP*, **0907** (2009) 009; *JHEP*, **0906** (2009) 041; *Phys. Rev. Lett.*, **101** (2008) 162001; S. Moch, P. Uwer and S. Weinzierl, *Nucl. Phys. Proc. Suppl.*, **116** (2003) 8

[13] R.K. Ellis, D.A. Ross and A.E. Terrano, *Nucl. Phys. B*, **178** (1981) 421

[14] E.A. Kuraev and V.S. Fadin, *Sov. J. Nucl. Phys.*, **1985** (41) 466; G. Altarelli and G. Martinelli, *Physics at LEP (J. Ellis and R. Peccei, eds.)*, **CERN Report 86-02 (Geneva, 1986)** (vol. 1) ; O. Nicrosini and L. Trentadue, *Phys. Lett. B*, **196** (1987) 551; *Z. Phys. C*, **39** (1988) 479

[15] G. Montagna, O. Nicrosini and F. Piccinini, *Phys. Lett. B*, **385** (1996) 348

[16] G. 't Hooft and M.J.G. Veltman, *Nucl. Phys. B*, **153** (1979) 365

[17] C. M. Carloni-Calame, S. Moretti, F. Piccinini and D. A. Ross, *Eur. Phys. J. C*, **62** (2009) 355; *Erratum-ibid. C*, **62** (2009) 453

[18] See, e.g.: Z. Kunszt and P. Nason, (conveners), in proceedings of the workshop 'Z Physics at LEP1' (G. Altarelli, R. Kleiss and C. Verzegnassi, eds.), preprint CERN-89-08, 21 September 1989 (and references therein).